MAE 533/601
Introduction to the Finite Element Method

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This course will provide a general preparation in computational solid mechanics for graduate engineering, science, and mathematics students who will pursue further work and research in specialized areas such as elasticity, plasticity, fracture mechanics, structural mechanics, mechanical design, fluid mechanics, and numerical analysis. Strongly recommended prerequisites: Advanced Strength of Materials, Elasticity, or Continuum Mechanics. Some of the basic topics to be covered will include:

**Week 1 An Introduction to the Use of Finite Element Procedures:** Physical problems and mathematical models; Applications of FEM in engineering and physics.

**Weeks 1-2 Basic Mathematical Methods:** Introduction to matrices; Vector spaces; Basic tensors.

**Weeks 3-5 Basic Concepts of Engineering Analysis and the Finite-Element Method:** Structural Matrices; Mathematical basis of FEM method and weak formulations; Discrete mathematical models: steady state solutions, eigenvalue problems; Continous system mathematical models: differential formulations, variational formulations, weighted residual methods, Ritz method, Galerkin formulations; Principle of Virtual Displacements; Energy methods; Imposition of constraints: Lagrange multipliers and Penalty methods.

**Weeks 6-8 Formulation of the FEM-Linear Analysis in Solid and Structural Mechanics:** Formulation of the Displacement-Based FEM: General derivation of finite-element equilibrium equations, imposition of displacement boundary conditions, generalized coordinate models, lumping of structure properties and loads; Convergence analysis of results: Criteria for monotonic convergence, properties of finite-element solutions, rate of convergence, calculation of stresses and the assessment of error; Incompatible and mixed FEM: Incompatible displacement based models, mixed formulations, mixed interpolation, incompressible analysis.

**Weeks 9-12 Formulation of Isoparametric Finite Element Matrices:** Isoparametric
derivation of stiffness matrix; Formulation of continuum elements: Quadrilateral elements, triangular elements; Convergence considerations; Element matrices in global coordinate system; Displacement/pressure based elements for incompressible media; Numerical integration: Interpolation polynomials, Newton-Cotes formulas, Gauss formulas; integrations in two and three dimensions; appropriate order of numerical integration. Modeling considerations

Weeks 13-14 Solution of Equilibrium Equations in Static Analysis: Direct Solutions: Gauss Elimination; Cholesky factorization; Computer implementation; Positive definiteness and Sturm sequence property; Iterative Solutions: Gauss Seidel Method and Conjugate Gradient methods with preconditioning; Solution Errors

Weeks 14-15 Finalization/Presentation of Projects/Paper Reviews

GRADING

Exams 50%
Two exams, each Exam 25%, these will be take-home exams

HW, Modeling Projects 50%

Knowledge of Fortran, C, C++, or other programming languages is helpful.

Class Notes, PDF files, and relevant papers will be handed out and will form the main core of the lectures. Lecture are on tape

Books on Reserve in Library

Concepts and Applications of Finite-Element Analysis by Cook, Malkus, Plesha, Witt: Will be available in the bookstore Other useful references
Finite-Element Procedures, Klaus-Jurgen Bathe
Finite-Element Programming, E. Hinton and D.R.J. Owen